JOURNAL OF AIRCRAFT
Vol. 29, No. 2, March–April 1992

# Integrated Aeroelastic Control Optimization of Laminated Composite Lifting Surfaces

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**An integrated design approach for aeroelastic control of laminated composite lifting surfaces is formulated and demonstrated. The design is posed as an optimization problem to determine the optimum ply orientation angles and the control law which maximizes the aeroelastic instability speed of an actively controlled laminated composite lifting surface subject to limited control authority and structural frequency constraints. The lifting surface is simulated by a rectangular symmetric cantilevered composite plate. The formulation incorporates the Rayleigh-Ritz energy method, two-dimensional incompressible unsteady aerodynamic theory, optimum control design, and parameter optimization techniques. The particular control configuration consists of two point-force inputs applied at the leading and trailing edges of the lifting surface. In conjunction with the laminated composite-plate structural model, the control configuration effectively simulates an active flexible lifting surface. The effectiveness of the integrated approach is evaluated by comparing the optimum designs obtained by the method with designs obtained by a design procedure using aeroelastic tailoring and optimum control design as well but in a nonintegrated fashion. The illustrative studies resulted in a design that exhibited 102% higher aeroelastic instability speed with 294.4% lower control cost than designs obtained by a nonintegrated design procedure.**

## Nomenclature

$A, B$ = $4n \times 4n$ dynamic matrix and $4n \times p$ control input distribution matrix, respectively, in the state space
$A_{CL}$ = closed-loop dynamic matrix
$A_4, A_5$ = $n \times n$ coefficient matrices of generalized circulatory aerodynamic mass and damping forces, respectively
$b$ = vector of design variables
$D_1, D_2$ = $n \times n$ coefficient matrices of induced aerodynamic forces
$F$ = $p$-component vector of control inputs
$f(b, g(b))$ = system-level objective function
$G$ = $p \times 4n$ linear state-feedback control gain matrix
$g$ = vector of control parameters
$I$ = $n \times n$ identity matrix
$J$ = control cost functional
$M, K$ = $n \times n$ structural mass and stiffness matrices, respectively
$M_a, C_a, K_a$ = $n \times n$ aerodynamic mass, damping, and stiffness matrices, respectively
$P$ = $4n \times 4n$ Riccati matrix
$Q, R$ = linear quadratic regulator state and control weighting matrices, respectively
$Q_a, Q_c$ = generalized unsteady aerodynamic and control forces, respectively
$q_i(t)$ = $i$th generalized coordinate
$V_{CR}, V_D, V_F$ = critical aeroelastic instability speed, divergence speed, and flutter speed, respectively
$V_{DES}$ = control design flight speed
$w(x, y, t)$ = out-of-plane deflection of lifting surface
$x(t)$ = $4n$ state vector
$,j$ = partial differentiation with respect to the $j$th design variable
$\Gamma$ = control input distribution matrix in configuration space
$\gamma_i(x, y)$ = $i$th assumed mode of deflection
$\eta$ = $n$-component vector of generalized aerodynamic lag states
$\theta$ = $l$-component vector of fiber orientations of individual laminae
$\rho, c, V$ = air density, chord length, freestream air velocity
$\phi(t)$ = Wagner's function
$\omega_i$ = $i$th structural frequency

Received April 25, 1989; presented at the VPI&SU/AIAA 7th Symposium on Dynamics and Control of Large Structures, Blacksburg, VA, May 8–10, 1989; revision received Jan. 17, 1991; accepted for publication Feb. 1, 1991. 

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## Introduction

THE advent of advanced fiber-reinforced composite materials has been called the biggest technical revolution since the jet engine.[1] Their high specific stiffness and strength properties have made possible the construction of very strong yet lightweight aircraft structures, which has resulted in substantial performance improvements. More importantly from the aeroelastic control point of view, these properties are directional. The directions of these properties determine the elastic deformation and thus the aeroelastic response of a composite lifting surface. The design process that seeks particular directional stiffness in aircraft structural design to control aeroelastic deformation of surfaces in order to enhance the aircraft's performance is known as aeroelastic tailoring.[2] Aeroelastic tailoring has been shown to be an effective means of passive control.[3–6] On the other hand, automatic control systems have been successfully used to actively control the aeroelastic response of aircraft in order to improve ride quality, reduce maneuver loads, and to suppress structural loads induced by atmospheric turbulence. In recent years, the feasibility and merits of designing automatic control systems to suppress aeroelastic instabilities have been established by a considerable amount of research.[7–12]

Passive control of aeroelastic behavior of a composite lifting surface can be achieved through aeroelastic tailoring by designing for the parameters of the laminated composite surface such as ply orientations, thicknesses, stacking sequences, etc. Active control can be exercised via automatic control systems by designing for the parameters of the control system such as the number and location of controls, which is the controller configuration, closed-loop poles, control gains, management of control authority, etc.

In recent years, there has been a surge of interest to abandon traditional compartmentalized disciplinary approaches to the design of flight vehicles and to integrate interests of different design groups in some optimum manner from a complex system's perspective. The first attempts at an interdisciplinary design approach have been in the area of structure and control design of flexible space vehicles. In this regard, Ref. 13 by Hale and Lisowski represents the first investigation that demonstrated the potential of an integrated (interdisciplinary) approach to design. Following the example of Ref. 13, a number of investigators[14–21] applied the concept of simultaneous structure and active control design to a variety of problems in spacecraft design each using different objective functions and constraint functions. The promise of application of integrated approach to spacecraft problems makes it also of interest for application to other areas of flight vehicle design. To this end, an integrated approach to aeroelastic control problems requiring interactions of aerodynamics, structures, and controls design groups stands out as an area in which benefits of an interdisciplinary design are highly desirable and critical to satisfaction of multidisciplinary design criteria for high-performance flight vehicles. Specifically in the area of advanced laminated composite lifting surfaces where the benefits of passive control via aeroelastic tailoring have already been demonstrated,[3–6] incorporation of active control design technology to the structural design of the lifting surface simultaneously to satisfy some flight performance objective at a system level would be of utmost interest.

Recently, Livne et al. in Ref. 22 addressed the multidisciplinary wing-synthesis problem presenting an integration framework for the methods of involved subdisciplines to design actively controlled fiber composite wings. Reference 22, however, presents no application of the ultimate goal of the subject and concludes that "integrated multidisciplinary optimization of practical actively controlled, fiber composite wings is within reach." Reference 23 also discusses, as one of the current activities at NASA Langley Research Center, the development of an integrated structure-control design capability, particularly utilizing laminated composite structures. Preceding Ref. 22, in Ref. 24, Zeiler and Weisshaar demonstrated the concept of integrated approach for an aeroservoelastic design problem. Reference 24 used a highly idealized aeroservoelastic model demonstrating "the benefits of" integrated design to "all participants." Reference 24 suggests that provisions for composite materials (and other issues) should certainly be pursued.

Although the activities cited by the above references were ongoing, the authors of this paper have also been involved in a comprehensive integrated approach to aeroelastic control problems specifically utilizing laminated composite lifting surfaces as structural models, thereby exploiting the promise of advances in composite structures technology. Following publication of Ref. 13 in 1983, the developments presented in this paper were initiated in 1984.[25] Reference 26 is a culmination of these efforts and the results reported in this paper and in Ref. 38 are excerpts from that investigation. The preliminary results of the work presented in this paper, however, were presented earlier in Ref. 27 which practically coincides with the publication of Ref. 24. Whereas the results reported in Ref. 24 and this paper have a common subject area, that both aspire to demonstrate the benefits of an integrated approach to an aeroservoelastic design problem, the developments presented in this work are completely independent of those of Ref. 24 and have no inspirational link to Ref. 24. However, from a general perspective, the particular structural model (laminated composite lifting surface) considered in this work represents a broader development and further verification of the benefits of integrated approach to an aeroelastic control problem as espoused in Ref. 24, and as aspired, in the conclusions of Refs. 22 and 23.

To the benefit of the reader, we shall describe briefly the technical differences between Ref. 24 and this investigation. "Composite materials are not expressly considered" in Ref. 24. A highly idealized aeroservoelastic system is used where two airfoils are rigidly attached to a rigid fuselage. The primary lifting surface models flexibility via a torsional and linear spring, whereas the second surface is assumed rigid. The model uses pitch and plunge degrees of freedom in addition to the angle of attack and control surface deflection. The system-level objective function is to minimize a particular stability margin subject to negativity constraint for a range of design speeds below an objective airspeed where design speeds are obtained by minimizing a quadratic control-performance measure. The structural design variables are the shear center and midchord locations. The control design speed is used as a control variable. Both the objective function and constraints are linear in design variables and a simplex-type procedure is used for optimization.

In this investigation, we are expressly concerned with the design of actively controlled laminated composite material lifting surfaces. The lifting surface is modeled as a laminated-composite cantilevered plate in an airstream. The geometry of the plate and the ply thicknesses are assumed constant although this is not necessarily needed. The ply orientation angles are considered as structural design variables, whereas the control design speed is used as a control variable. The system-level objective is to maximize the aeroelastic instability (divergence/flutter) speed subject to a control authority constraint limiting the minimum value of a control-performance measure obtained via the linear quadratic regulator theory.[29] In addition, at the structure level, lower bounds on the natural frequencies of the composite lifting surface are considered along with the size constraints on the structural design variables. The system-level, control-level, and structure-level functions are nonlinear in design variables so that our optimization procedure employed two sophisticated widely used optimization routines, I-DESIGN[30] by Arora et al. and NEWSUMT-A[31] by Haftka et al. also providing a comparison of optimum solutions. Extensive computer-code development and integration of modular disciplinary codes were undertaken to produce the results reported in this paper.

Perhaps equally worth noting in this study is the application of the integrated design approach to a lifting surface with an unconventional control configuration, in that the method is illustrated by using two point-force inputs located at the leading and trailing edges, instead of the conventional control surfaces. Thus, the particular controller configuration interacts completely with the structural flexibility to control the aeroelastic phenomenon whereby the aerodynamics is indirectly controlled via structural flexibility. Such a structure-control model is akin to the active flexible wing (AFW) concept. The concept of AFW is under current investigation by NASA Langley Research Center and Rockwell.[32,33] Making the wing flexibility a fuller participant in the improvement of aeroelastic performance adds a new dimension to such designs. In contrast, traditional control surfaces primarily act to control the unsteady aerodynamics whereby structural flexibility is controlled indirectly via aerodynamic changes.

The paper is organized as follows. First, the aeroelastic mathematical model is developed and the optimal control law is presented for active flutter suppression. Subsequently, the integrated aeroelastic control optimization problem is formulated. Two illustrative examples are then given for tailoring the layer orientations of an actively controlled composite lifting surface. In the first illustration an active control design speed is prescribed for optimum tailoring, whereas in the second illustration control design speed is treated as a design parameter as well in addition to the layer orientations. Both quantitative and qualitative aspects of the integrated design are discussed following the illustrative examples. Finally, conclusions and further developments are given.

## Mathematical Model

The composite wing is idealized by a rectangular, symmetric laminated plate. The control system is represented by discrete forces acting perpendicular to the plate's surface (see Fig. 1). The plate is in a uniform incompressible airstream of velocity $V$ in the direction of the positive $y$ axis. Since the laminated plate is symmetric about the midplane, the in-plane and out-of-plane motions are decoupled. For aeroelastic purposes, only the out-of-plane motion is of interest. Assuming linear aeroelasticity, the equations of motion of the actively controlled composite plate are obtained by the Rayleigh-Ritz energy method. The out-of-plane deflection of the plate in generalized coordinates is expanded in terms of assumed modes:

$$w(x, y, t) = \sum_{i=1}^{n} \gamma_i(x, y)q_i(t) \tag{2.1}$$

Using the Rayleigh-Ritz method, the aeroelastic equations of motion of the system are obtained in the form

$$M\ddot{q} + K(\theta)q = Q_a + Q_c \tag{2.2}$$

The expression for the generalized unsteady aerodynamic forces has the form

$$Q_a(t) = \frac{\pi\rho c^2}{4} M_a\ddot{q} + \pi\rho c V C_a\dot{q} + \pi\rho V^2 K_a q$$
$$+ \pi\rho c V D_1\eta_1 + \pi\rho c V D_2\eta_2 \tag{2.3}$$

Introduction of aerodynamic lag states $\eta_1$ and $\eta_2$ replaces the Duhamel integrals involved in the inverse Fourier transformation of the circulatory aerodynamic force terms. The development of Eq. (2.2) along with the mass, stiffness, and the aerodynamic matrices in Eq. (2.3) is presented in detail in Ref. 26. Specifically, if Wagner's function $\phi(t)$ appearing in the Duhamel's integral is approximated by W. P. Jones' expression[34] for subsonic speeds

$$\phi(t) = 1 - 0.165e^{-0.041t} - 0.335e^{-0.32t}$$

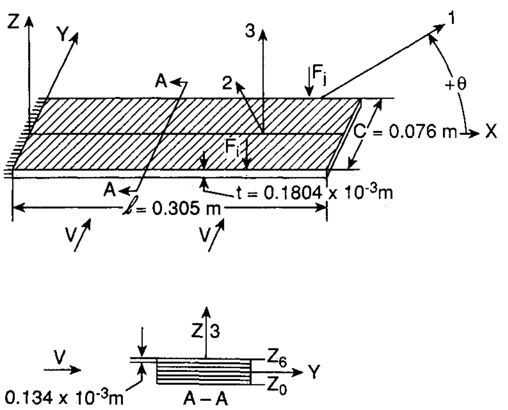


**Fig. 1 Dimensions, fiber orientation convention, and structural axes of actively controlled composite plate in an airstream.**

then as shown by Rodden and Stahl,[35] the Duhamel's integral can be replaced by the first-order aerodynamic lag state equations:

$$\dot{\eta}_1 = -0.041\frac{2V}{c}I\eta_1 - A_4\ddot{q} + \frac{V}{c}A_5\dot{q} \tag{2.4a}$$

$$\dot{\eta}_2 = -0.32\frac{2V}{c}I\eta_2 - A_4\ddot{q} + \frac{V}{c}A_5\dot{q} \tag{2.4b}$$

yielding $D_1 = -0.165I$ and $D_2 = -0.335I$ in Eq. (2.3). Equations (2.4) in conjuction with Eqs. (2.2) and (2.3) complete the aeroelastic dynamics.

Lastly, the expression of the generalized control forces can be rewritten as

$$Q_c(t) = \Gamma^T F(t) \tag{2.5}$$

Placing the expressions of the generalized aerodynamic and control forces from Eqs. (2.3) and (2.5) into Eq. (2.2), the aeroelastic equations of motion in the configuration space become

$$\left[M - \frac{\pi\rho c^2}{4}M_a\right]\ddot{q} - \pi\rho c V C_a\dot{q} + [K(\theta) - \pi\rho V^2 K_a]q$$
$$- \pi\rho c V D_1\eta_1 - \pi\rho c V D_2\eta_2 = \Gamma^T F \tag{2.6}$$

Next, defining the state variable vector of the system as

$$x(t) = [\dot{q}^T\ q^T\ \eta_1^T\ \eta_2^T]^T \tag{2.7}$$

Equations (2.4) and (2.6) are combined in the state-space form:

$$\dot{x} = A(\theta, V)x + BF \tag{2.8}$$

Although in the derivation of Eq. (2.8), $2n$ states were assumed for the aerodynamic lag dynamics, this is not necessarily needed, and a reduced number of aerodynamic states can be used via minimum state methods to represent the lag dynamics.[22-23]

The control input $F$ is assumed as a linear state feedback law given by

$$F = -G(\theta, V_{DES})x \tag{2.9}$$

The control design speed $V_{DES}$ is used to determine the control law $F$ in the following way. The aeroelastic motion of a particular laminate design $\theta$ at flight speed $V = V_{DES}$ is considered:

$$\dot{x} = A(\theta, V_{DES})x + BF \tag{2.10}$$

The control law of Eq. (2.9) can be obtained through the minimization of the cost functional:

$$2J(\theta, V_{DES}) = \int_0^{\infty} (x^T Q x + F^T R F)\, dt \tag{2.11}$$

associated with the motion of the system determined by Eq. (2.10). For simplicity, the weighting matrices $Q$ and $R$ are chosen as identity matrices in this paper.

The control input $F$ that minimizes Eq. (2.11) is given by[29]

$$F = -R^{-1}BP(\theta, V_{DES})x = -G(\theta, V_{DES})x \tag{2.12}$$

in which for $P$ we consider the associated steady-state Riccati equation:

$$PA + A^T P - PBR^{-1}B^T P + Q = 0 \tag{2.13}$$

The resulting minimum value of the cost functional is given by

$$2J = x_0^T P(\theta, V_{DES}) x_0 \tag{2.14}$$

where $x_0$ is an initial state disturbance. The cost functional $J$ will be loosely referred to herein as the control cost. Once the control law is determined, the closed-loop state-space equations of the particular laminate are only functions of time and airspeed $V$. Then the closed-loop state equations become

$$\dot{x} = A_{CL}(\theta, g(\theta, V_{DES}), V)x, \qquad A_{CL} = A - BG \tag{2.15}$$

where $g$ is the control parameters vector, consisting of the elements of the control gain matrix $G$.

The aeroelastic analysis with active controls is performed by stability analysis of the closed-loop system equations. For a particular laminate design $\theta$ and control design speed $V_{DES}$, the closed-loop system matrix $A_{CL}$ is only a function of the airspeed $V$. The stability of the aeroelastic response, at a particular airspeed $V$, is determined by solving the eigenvalue problem of the closed-loop matrix $A_{CL}$. An airspeed that renders the matrix $A_{CL}$ neutrally stable is referred to as the critical aeroelastic instability speed and denoted by $V_{CR}$. Specifically, the flight speed that corresponds to flutter is denoted by $V_F$, whereas the speed that corresponds to divergence instability is denoted by $V_D$. In the case of divergence, the critical speed is corrected to account for the effects of finite span by applying the correction suggested in Ref. 28.

The specific structure to which the above formulation was applied had six graphite/epoxy plies with length, width, and ply thickness of 0.305, 0.076, and $0.134 \times 10^{-3}$ m, respectively. A five-term expansion, $n = 5$, was used in Eq. (2.1). The assumed modes $\gamma_i$ ($i = 1, 5$) represented the first and second spanwise bending taken as the modes of a cantilevered beam, first and second torsion modes of decoupled torsional motion of a cantilevered plate, and the first chordwise flexible mode providing a parabolic change of the camber of the mean chord. The aeroelastic instabilities of the particular geometry and modal representation of the plate were studied analytically and experimentally by Hollowell and Dugundji.[28] However, the last three assumed modes of the structural model used in this study are different than those in Ref. 28 in that these modes satisfy the required geometric boundary conditions in our model, whereas in Ref. 28 they do not. Consequently, our analytical results agreed better with the experimental results in Ref. 28 than did the analytical results of Ref. 28. Having verified the fidelity of our structural model and thus the computer-code development, the integrated design of active control and tailoring of the composite plate was undertaken as formulated in the next section. For the control configuration, two point-force actuators, $p = 2$, represented by $F_i$ and $F_j$, were used at $\frac{3}{4}$ of span at the leading and trailing edges as shown on Fig. 1.

## Formulation of an Integrated Aeroelastic Control Optimization

The external forces considered here are the aerodynamic and control forces which, as can be seen from Eqs. (2.3) and (2.5) in conjunction with Eq. (2.12), are functions of the plate's elastic deformation, control design, and the airspeed of the flow. Thus, the aeroelastic response as well as the performance of the active control system are determined by the interaction of laminate and control design. Using control systems terminology, the effective aeroelastic control law is the combination of the active control law and the passive control law embedded in the structure in the form of flexural moduli, by the laminate design. It follows that in order to achieve effective aeroelastic control, the design process must jointly consider laminate and active control design.

The aeroelastic control design is posed as a multilevel optimization problem. The system-level objective function to be maximized is the critical aeroelastic speed $V_{CR}$ of the composite lifting surface, while at the same time the control cost $J$ is minimized at the control subsystem level. The design variables of the problem are the fiber orientations $\theta$ of the individual laminae of the laminate and the control design speed $V_{DES}$.

In a nested optimization procedure, an upper bound on the minimum control cost $J$ is placed, indicating limited control authority. Structural constraints are imposed in the form of upper or lower bounds on the natural vibration frequencies of the plate.

The optimization problem is stated as: determine the design variables $b$ to

$$\text{Maximize } V_{CR} = f(b, g(b)) \tag{3.1}$$

subject to the control authority constraint

$$J - \hat{J} \leq 0 \tag{3.2a}$$

and structural frequency constraints

$$\hat{\omega}_i - \omega_i \leq 0, \qquad i = 1, 2, \ldots \tag{3.3}$$

and the size bounds on the design variables

$$\hat{b}_L < b < \hat{b}_U$$

where $b = [\theta^T \, V_{DES}]^T$ and $(\hat{\ })$ denotes constraint values. Equation (3.1) represents implicit functional dependence of the critical speed in terms of the design variables. Inequality (3.2a) implies

$$x_0^T(P - \hat{P})x_0 \leq 0 \qquad \text{or} \qquad x_0^T \Delta P x_0 \leq 0 \tag{3.2b}$$

where $\Delta P = P - \hat{P}$ is sign definite, and thus the constraint (3.2a) can be substituted with

$$\rho_1 \leq 0 \tag{3.2c}$$

where $\rho_1$ is the largest eigenvalue of $\Delta P$. The form of inequality constraint (3.2b) also represents a control-performance robustness bound.[36] The optimization problem given by Eqs. (3.1–3.3) is only representative of an interdisciplinary formulation and by no means unique. Certainly the variety of realistic formulations is only restricted by the innovativeness of the designer; this in itself may represent a challenge depending on the complexity and the nature of interdisciplinary interactions.

For optimization purposes, the gradients of the constraint functions and the objective function are required. These can be obtained either analytically or numerically by finite differencing. However, whenever possible analytical expressions are desirable. The sensitivity coefficients of the control constraint with respect to the $j$th design variable is given by

$$\rho_{1,j} = y_1^T P_{,j} y_1, \qquad P_{,j} = \frac{\partial P}{\partial b_j} \tag{3.4}$$

where $y_1$ is the eigenvector of $\Delta P$ that corresponds to $\rho_1$. The sensitivity of the Riccati matrix $P$ to the $j$th design variable is obtained by differentiating Eq. (2.13) and solving the resulting algebraic Lyapunov equation.[37]

$$P_{,j} A_{CL} + A_{CL}^T P_{,j} = -(P A_{CL,j} + A_{CL,j}^T P^T) \tag{3.5}$$

The sensitivity coefficients of the natural frequencies with respect to the $j$th design variable are given by

$$\omega_{i,j}^2 = x_i^T K_{,j} x_i \tag{3.6}$$

where $x_i$ is the corresponding modal vector. In contrast to the gradients of the constraint functions given by Eqs. (3.5) and (3.6), it is not feasible to generate an explicit expression for the critical speed $V_{CR}$ as the objective function. This follows from the fact that the objective function $V_{CR}$ is associated with the special, imaginary roots of the characteristic equation of the closed-loop matrix $A_{CL}$. Thus, an expression for the sensitivity coefficients $\partial V_{CR}/\partial b_i$ will involve the root sensitivity of an equation which in turn will require evaluation of derivatives of many combinations of subdeterminants of $A_{CL}$ with respect to the design variables. Although the procedure can be formulated symbolically, it can only effectively be executed on the computer. In fact, this process is a matter of cybernetics, and the sensitivity coefficients $\partial V_{CR}/\partial b_i$ are obtained in this study numerically by finite differencing the (flutter/divergence) solutions of the characteristic equation with respect to the design variables. The programming structure of the method outlined above is given in Fig. 2.

## Illustrative Examples

Two different design problems are considered. In the first, the control design speed has a prescribed value, whereas in the second problem the control design speed is considered to be a design variable also. The optimum aeroelastic control law will consist of the laminate design and the optimum active control law that together maximize the critical speed of the aeroelastic system.

Numerical solutions to the two problems have been obtained by using the IDESIGN[30] and NEWSUMT-A[31] optimization programs. IDESIGN is based on a combination of cost bounding and recursive quadratic algorithms, whereas NEWSUMT-A is based on a sequential unconstrained minimization algorithm using Newton's method with approximate derivatives.

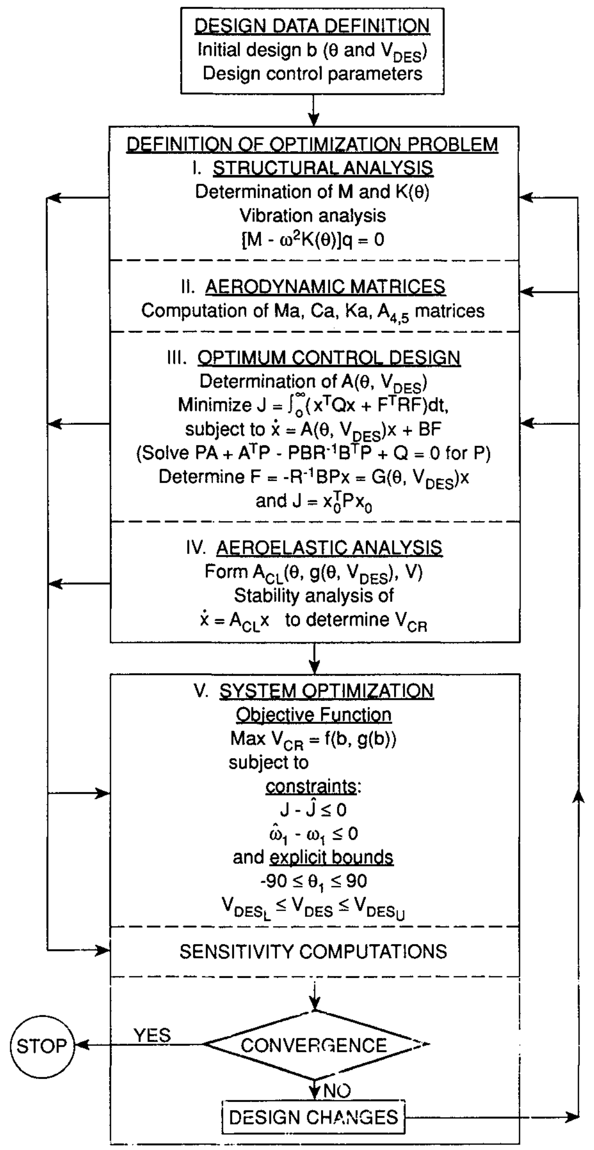

**Fig. 2 Integrated aeroelastic optimization programming structure.**

### Optimum Aeroelastic Control Design with Prescribed Control Design Speed

In this problem, the control design speed of the active aeroelastic control system is prescribed to be 45 m/s. At this airspeed, the uncontrolled aeroelastic response of the composite plate is unstable regardless of laminate design. Thus, an active aeroelastic control system is required to render the aeroelastic response stable at that airspeed.

The control authority of the active aeroelastic control system is constrained by a realistic value of $\hat{J} = 4.5$. The value of $\hat{J}$ was chosen based on the resulting control costs of actively controlled several plate configurations with different layer orientations through a range of control design speeds from 5–150 m/s for each layer configuration. These results are documented in Ref. 26. Upon observing the order of magnitudes of control costs for this ensemble of nonintegrated designs, a constraint value of $\hat{J} = 4.5$ was deemed realistic. In addition, one structural constraint, $\hat{\omega}_1 = 4.5$ Hz, is imposed as a lower bound in the first natural vibration frequency of the plate. The particular lower bound on the fundamental frequency of the plate was chosen after observing the fundamental frequencies of a number of layer configurations including optimal layer configurations obtained by passive (no control law) aeroelastic tailoring via maximization of critical aeroelastic instability speed in Ref. 26. The value of $\hat{\omega} = 4.5$ was deemed realistic as well. The specification of lower bound on $\omega$ reflects a desire on minimum stiffness of the plate to increase aeroelastic instability speeds passively. From a control design point of view, such lower bounds on uncontrolled structural frequencies also tend to lower the control cost. Consequently, the structural frequency contraint indirectly interacts with the control cost contraint making the satisfaction of the control cost constraint easier. The discussion of such aspects for structural control problems in general is given in Ref. 21.

The results are presented in Tables 1 and 2. The initial designs were chosen so that the corresponding plates had distinctively different flexural moduli. The flexural moduli and the natural frequencies of the initial laminates and the locally optimum laminates are given in Ref. 26.

The design with the highest critical aeroelastic airspeed $V^*_{CR}$ among the locally optimum designs was considered to be the optimum design. That was the laminate design $[-8.1/16.6/-17.9]^*_s$ together with the optimum active control law, which has a control cost of $J^* = 2.84$. The optimum design exhibited divergence instability at $V^*_{CR} = V_D = 80.6$ m/s.

### Optimum Aeroelastic Control Design with the Control Design Speed as a Design Variable

In this problem, the control design speed $V_{DES}$ is considered to be a design variable allowed to vary from 40–150 m/s. The upper bound on the control constraint was set equal to the optimum control cost determined in the first problem: that is $\hat{J} = 2.84$.

Treating $V_{DES}$ as a design variable increases the flexibility of the design process and the feasible design area considerably. This can be seen from the fact that when $V_{DES}$ is considered constant, then for a particular laminate design $\theta$, a unique active control law is determined by Eq. (2.9). However, when $V_{DES}$ is a design variable, then for a particular laminate design $\theta$, a class of admissible active control laws is

**Table 1 Optimum aeroelastic control design via IDESIGN with prescribed control design speed[a]**

| Initial design | Initial control cost | Initial critical speed | Locally optimum design | Locally optimum control cost | Locally optimum critical speed | Percent increase in critical speed | Percent decrease in control cost |
|---|---|---|---|---|---|---|---|
| $\theta^0$ | $J^0$ | $V^0_{CR}$, m/s | $\theta^*$ | $J^*$ | $V^*_{CR}$, m/s | $V_{CR}$, % | $J$, % |
| $[-45.0/-45.0/-45.0]_s$ | 8.21 | $V_F = 61.69$<br>$f_F = 28.01$ | $[0.777/-14.212/-29.874]_s$ | 2.67 | $V_D = 79.59$ | 29.02 | 67.48 |
| $[45.0/-45.0/0.0]_s$ | 9.59 | $V_F = 65.03$<br>$f_F = 31.23$ | $[-0.451/-16.006/8.185]_s$ | 2.61 | $V_D = 79.28$ | 21.91 | 72.78 |
| $[45.0/45.0/45.0]_s$ | 10.83 | $V_F = 69.15$<br>$f_F = 31.02$ | $[3.210/20.123/41.330]_s$ | 4.03 | $V_F = 76.33$<br>$f_F = 113.91$ | 10.38 | 62.79 |
| $[0.0/0.0/0.0]_s$ | 2.78 | $V_D = 65.76$ | $[-8.096/12.525/-11.684]_s$ | 2.71 | $V_D = 80.22$ | 21.99 | 2.52 |
| $[-70.0/70.0/0.0]_s$ | 17.35 | $V_F = 71.20$<br>$f_F = 18.30$ | $[3.029/20.433/33.711]_s$ | 3.96 | $V_F = 76.07$<br>$f_F = 113.83$ | 6.84 | 77.18 |

[a]$V_{DES} = 45$ m/s, $\hat{J} = 4.5$, $\hat{\omega}_1 = 4.5$ Hz, $b = [\theta_1\ \theta_2\ \theta_3]^T$, $f$ in Hz.

**Table 2 Optimum aeroelastic control design via NEWSUMT-A with prescribed control design speed[a]**

| Initial design | Initial control cost | Initial critical speed | Locally optimum design | Locally optimum control cost | Locally optimum critical speed | Percent increase in critical speed | Percent decrease in control cost |
|---|---|---|---|---|---|---|---|
| $\theta^0$ | $J^0$ | $V^0_{CR}$, m/s | $\theta^*$ | $J^*$ | $V^*_{CR}$, m/s | $V_{Cr}$, % | $J$, % |
| $[-45.0/-45.0/-45.0]_s$ | 8.21 | $V_F = 61.69$<br>$f_F = 28.01$ | $[-8.106/16.597/-17.902]_s$ | 2.84 | $V_D = 80.60$ | 30.65 | 65.41 |
| $[45.0/-45.0/0.0]_s$ | 9.59 | $V_F = 65.03$<br>$f_F = 31.23$ | $[33.350/-16.679/89.790]_s$ | 6.74 | $V_F = 67.00$<br>$f_F = 30.04$ | 3.03 | 29.72 |
| [45.0/45.0/45.0]/zs | 10.83 | $V_F = 69.15$<br>$f_F = 31.03$ | $[-8.489/12.874/-63.008]_s$ | 3.18 | $V_D = 80.34$ | 16.18 | 70.64 |
| $[0.0/0.0/0.0]_s$ | 2.78 | $V_D = 65.76$ | $[-6.542/-3.287/2.409]_s$ | 2.43 | $V_D = 76.75$ | 16.71 | 12.59 |
| $[-70.0/70.0/0.0]_s$ | 17.35 | $V_F = 71.20$<br>$f_F = 18.30$ | $[-8.806/12.224/88.499]_s$ | 3.40 | $V_D = 80.51$ | 13.08 | 80.40 |

[a]$V_{DES} = 45$ m/s, $\hat{J} = 4.5$, $\hat{\omega}_1 = 4.5$ Hz, $b = [\theta_1\ \theta_2\ \theta_3]^T$, $f$ in Hz.

**Table 3 Optimum aeroelastic control design via IDESIGN with control design speed as a variable[a]**

| Initial design | Initial control cost | Initial critical speed | Locally optimum design | Locally optimum control cost | Locally optimum critical speed | Percent increase in critical speed | Percent decrease in control cost |
|---|---|---|---|---|---|---|---|
| $\theta^0$, $V^0_{DES}$, m/s | $J^0$ | $V^0_{CR}$, m/s | $\theta^*$, $V^*_{DES}$, m/s | $J^*$ | $V^*_{CR}$, m/s | $V_{CR}$, % | $J$, % |
| $[-45.0/-45.0/-45.0]_s$<br>$V_{DES} = 45.0$ | 8.21 | $V_F = 61.69$<br>$f_F = 28.02$ | $[2.612/-25.248/-25.253]_s$<br>$V_{DES} = 76.72$ | 2.84 | $V_D = 99.88$ | 61.9 | 65.4 |
| $[45.0/-45.0/0.0]_s$<br>$V_{DES} = 40.0$ | 9.88 | $V_F = 59.11$<br>$f_F = 30.81$ | $[-18.747/12.264/-18.746]_s$<br>$V_{DES} = 98.69$ | 2.84 | $V_D = 112.98$ | 91.1 | 71.3 |
| $[45.0/45.0/45.0]_s$<br>$V_{DES} = 40.0$ | 10.83 | $V_F = 69.15$<br>$f_F = 31.03$ | $[-19.301/-19.296/-19.304]_s$<br>$V_{DES} = 95.16$ | 2.84 | $V_D = 106.70$ | 54.3 | 73.8 |
| $[0.0/0.0/0.0]_s$<br>$V_{DES} = 40.0$ | 2.86 | $V_D = 56.59$ | $[-18.739/12.279/-18.809]_s$<br>$V_{DES} = 98.69$ | 2.84 | $V_D = 112.98$ | 99.6 | 0.7 |
| $[-70.0/70.0/0.0]_s$<br>$V_{DES} = 40.0$ | 17.56 | $V_F = 62.97$<br>$f_F = 20.82$ | $[-18.748/12.263/-18.739]_s$<br>$V_{DES} = 98.69$ | 2.84 | $V_D = 112.98$ | 79.4 | 83.8 |

[a]$\hat{J} = 2.84$, $\hat{\omega}_1 = 4.5$ Hz, $b = [\theta_1\ \theta_2\ \theta_3\ V_{DES}]^T$, $f_F$ in Hz.

**Table 4 Optimum aeroelastic control design via NEWSUMT-A with the control design speed as a variable[a]**

| Initial design | Initial control cost | Initial critical speed | Locally optimum design | Locally optimum control cost | Locally optimum critical speed | Percent increase in critical speed | Percent decrease in control cost |
|---|---|---|---|---|---|---|---|
| $\theta^0$, $V^0_{DES}$, m/s | $J^0$ | $V^0_{CR}$, m/s | $\theta^*$ $V^*_{DES}$, m/s | $J^*$ | $V^*_{CR}$, m/s | $V_{CR}$, % | $J$, % |
| $[-45.0/-45.0/-45.0]_s$<br>$V_{DES} = 45.0$ | 8.21 | $V_F = 61.69$<br>$f_f = 28.02$ | $[-35.408/-8.342/-9.026]_s$<br>$V_{DES} = 82.57$ | 4.14 | $V_D = 102.68$ | 66.4 | 49.6 |
| $[45.0/-45.0/0.0]_s$<br>$V_{DES} = 40.0$ | 9.88 | $V_F = 59.11$<br>$f_F = 30.81$ | $[15.415/-19.798/89.795]_s$<br>$V_{DES} = 65.49$ | 4.28 | $V_F = 90.66$<br>$f_F = 122.20$ | 53.4 | 56.7 |
| $[45.0/45.0/45.0]_s$<br>$V_{DES} = 45.0$ | 10.83 | $V_F = 69.15$<br>$f_F = 31.03$ | $[-18.308/-20.281/-22.190]_s$<br>$V_{DES} = 94.85$ | 2.82 | $V_D = 106.25$ | 53.7 | 74.0 |
| $[0.0/0.0/0.0]_s$<br>$V_{DES} = 40.0$ | 2.86 | $V_D = 56.59$ | $[-19.587/-15.402/24.583]_s$<br>$V_{DES} = 96.29$ | 2.83 | $V_D = 108.44$ | 91.6 | 1.0 |
| $[-70.0/70.0/0.0]_s$<br>$V_{DES} = 40.0$ | 17.56 | $V_F = 62.97$<br>$f_F = 20.82$ | $[-18.856/11.691/-19.261]_s$<br>$V_{DES} = 98.50$ | 2.83 | $V_D = 112.69$ | 79.0 | 83.9 |

[a]$\hat{J} = 2.84$, $\hat{\omega}_1 = 4.5$ Hz, $b = [\theta_1\ \theta_2\ \theta_3\ V_{DES}]^T$, $f_F$ in Hz.

determined by Eq. (2.9). Therefore, by considering $V_{DES}$ as a design variable, the quality of the optimum solution is expected to increase. This of course can be shown by comparing the solutions of the two design problems presented here.

The results are presented in Tables 3 and 4. As in the previous problem, the design with the highest critical aeroelastic speed $V_{CR}^*$ among the locally optimum designs was considered to be the optimum design. The optimum design was determined to be the $[-18.7/12.3/-18.7]_s^*$ laminate design together with the optimum active control law of control design speed $V_{DES}^* = 98.69$ m/s and control cost $J^* = 2.84$. This optimum design exhibited divergence instability at $V_{CR}^* = V_D = 112.98$ m/s.

The flexural moduli and the natural frequencies of the locally optimum laminates are given in Ref. 26.

Comparing the results of this design problem with those of the previous one, it can be seen that considering the control design speed to be a design variable resulted in an optimum design with critical speed 40.17% higher than that of the optimum design obtained considering $V_{DES}$ to be a constant, with no increase in control cost.

It is also noted that the optimum design of this problem was the solution of three design cases with distinctly different initial designs using IDESIGN, as shown in Table 3. A very similar design was also obtained by NEWSUMT-A, as shown in the last row of Table 4.

Having obtained numerical solutions to these two design problems, it must be mentioned that solutions obtained by NEWSUMT-A in three cases stopped in the infeasible design area, violating the control constraint, as can be seen in Tables 2 and 4.

## Discussion and Evaluation of Integrated Approach

Examining the data presented in Tables 1–4 and the flexural moduli and natural frequencies of the optimum laminate designs reported in Ref. 26, the following observations are made. First, all of the laminates of the locally optimum designs had high bending stiffnesses. This is a reflection of the fact that the composite plate must be sufficiently stiff in bending in order to accommodate the particular active control system. The aeroelastic tailoring proces of the method provides laminate designs consistent with the structural constraints and compatible with the active control law.

Secondly, the overwhelming majority of the locally optimum designs were wash-in[6,26] laminates and the aeroelastic instabilities were divergence. The remaining locally optimum designs were moderate wash-out[6,26] laminates and the aeroelastic instabilities were bending-torsion flutter. This observation is consistent with the analysis in Ref. 26, which concluded that the wash-in laminates can be more effectively controlled with the active control system considered in this paper.

Thirdly, although the majority of the configurations of the laminates of the locally optimum designs were distinct, their flexural moduli and natural frequencies were not. In particular, the flexural moduli and natural frequencies as well as the critical aeroelastic speeds of the optimum laminate configurations appearing in the first, second, and fourth rows of Table 1 have approximately the same numerical values, and they are all wash-in laminates. The same is true for the laminates in the third and fifth rows of Table 1, with the exception that they are wash-out laminates. The same observations are made from Table 2 where the optimum configurations of the first, third, fourth, and fifth rows are wash-in laminates and have approximately the same flexural moduli, natural frequencies, and critical aeroelastic speeds.

The laminates of the locally optimum designs in Table 3 are all wash-in laminates. The laminates of the second, fourth, and fifth rows of the same table have almost identical configurations, flexural moduli, natural frequencies, control design speeds, and critical aeroelastic speeds. The flexural moduli and the natural frequencies of the laminates in the first and third rows of Table 3 have nearly the same values as other laminates, although the control design speeds and thus the critical aeroelastic speeds are distinct. The above pattern is repeated for the optimum solutions in Table 4.

The above observations concerning the results of this investigation are a reflection of the fact that a number of different laminate configurations of the same plate may have the same constitutive relations and therefore the same dynamic and aeroelastic behavior. It can therefore be concluded that the flexural moduli can be a more suitable set of design variables for determining the passive aeroelastic control law, if an explicit laminate configuration is not required. This conclusion is consistent with the approach taken in Ref. 6.

In order to evaluate the effectiveness of the integrated aeroelastic control optimization method, the optimum designs of the two design problems presented above are compared to a design obtained by a nonintegrated design procedure. This nonintegrated design procedure consists of aeroelastic tailoring and optimum control design as well, but the two design processes are performed independently. Specifically, the aeroelastic control design is determined by tailoring the composite plate for maximum critical aeroelastic speed, and then an optimum active control law with prescribed control design speed is determined for the same two-input control system considered throughout this study.

The prescribed control design speed was chosen to be $V_{DES} = 45.0$ m/s so that direct comparison of the resulting design with the optimum design obtained with the prescribed control design speed is possible. The comparison is extended to include the results of the optimum design obtained with the variable control design speed, since the control cost for the optimum designs of both problems is the same.

The laminate design of the composite plate that exhibits the maximum uncontrolled critical aeroelastic speed has been determined in Ref. 26 through aeroelastic tailoring. The optimum configuration was $[-55.66/43.62/4.18]_s^*$, which exhibited divergence instability at $V_{CR}^* = V_D = 43.29$ m/s.

For this optimum configuration and control design speed $V_{DES} = 45.0$ m/s, an optimum active control law is determined through the minimization of the quadratic control cost functional given by Eq. (2.11). The optimum active control law is given by Eq. (2.12) and its application improved the aeroelastic performance of the plate, rendering $V_C^* = V_D = 55.92$ m/s with control cost $J^* = 11.2$. The results of the two design problems using the integrated aeroelastic optimization method and of the nonintegrated design procedure are presented in Table 5 for comparison. It can be seen that the integrated aeroelastic control design method resulted in designs with substantially higher critical aeroelastic speed and lower control cost.

When the method was applied with constant control design speed $V_{DES} = 45.0$ m/s, the resulting optimum design exhibited a 44.13% higher critical speed with 294.4% lower control cost than the design obtained using the nonintegrated design procedure. Treating the control design speed $V_{DES}$ as a design variable, the integrated aeroelastic control optimization yielded

**Table 5 Comparison of aeroelastic control design performances**

| Method | Laminate design $\theta$ | $V_{CR}$, m/s | $J$ |
|---|---|---|---|
| Nonintegrated aeroelastic control design. $V_{DES} = 45.0$ m/s | $[-55.66/43.63/4.18]_s$ | $V_D = 55.92$ | 11.2 |
| Integrated aeroelastic control design. $V_{DES} = 45.0$ m/s | $[-8.1/16.6/-17.9]_s$ | $V_D = 80.6$ | 2.84 |
| Integrated aeroelastic control design, with $V_{DES}$ as a control design variable | $[-18.7/12.3/-18.7]_s$ | $V_D = 112.98$ | 2.84 |

a further improved design. The resulting optimum design exhibits 40.2% higher critical aeroelastic speed than the optimum design obtained using the method with the control design speed constant, with the same control cost, and 102% higher critical aeroelastic speed than the design obtained by the nonintegrated procedure.

## Conclusions and Further Developments

The integrated aeroelastic control optimization method demonstrated in this paper resulted in designs with the best combination of laminate and control system design, yielding superior aeroelastic performance at the expense of lower control cost.

One of the most important features of the integrated aeroelastic control optimization method is its adaptability to improvements in the technologies involved and to the degree of complexity of the design problem. The aeroelastic mathematical model, active control design, and optimization algorithm have all been developed in modular form and therefore can be modified to improve the efficiency and effectiveness of the method. Additional design variables, such as the number of plies, sweep angle, and aspect ratio of the composite plate, as well as different types of fibrous composites, can be easily incorporated into the present formulation with minor modeling modifications. For a more realistic wing model, the aeroelastic mathematical model can be formulated using more elaborate, advanced theories, such as finite elements and a three-dimensional, unsteady, compressible lifting surface aerodynamic theory. Rigid-body modes, simulating aircraft motion and thermal effects due to large temperature gradients in the structure predicted at high flight speeds, can be incorporated into the formulation.

Other, more conventional forms of active aeroelastic control such as rapidly responding control surfaces can be considered. Different control system-performance measures, such as the Frobenius norm of $G^TRG$,[20] and different optimum control design methods can be used. The use of application-specific, improved optimization algorithms can greatly improve the efficiency and effectiveness of the integrated aeroelastic control method.

The method has been demonstrated to be a versatile, powerful procedure for aeroelastic control design of laminated composite lifting surfaces. Because of its control outlook and methodology, the method can become an integral part of control-configured design procedures to design efficient aerospace vehicles.

## Acknowledgment

This research was supported in part by The Ohio State University Seed Grant Program.

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